

POPULATION BASED ANT COLONY OPTIMIZATION FOR INVENTORY ROUTING PROBLEM

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ABSTRACT

The inventory routing problem presented in this study is a one-to-many distribution network consisting of a manufacturer that produces multi products to be transported to many geographically dispersed customers. We consider a finite horizon where a fleet of capacitated homogeneous vehicles, housed at a depot/warehouse, transports products from the warehouse to meet the demand specified by the customers in each period. The demand for each product is deterministic and time varying and each customer request a distinct product. The inventory holding cost is product specific and is incurred at the customer sites. The objective is to determine the amount on inventory and to construct a delivery schedule that minimizes both the total transportation and inventory holding cost while ensuring each customer's demand is met over the planning horizon. The problem is formulated as a mixed integer programming problem and is solved using CPLEX to get the lower bound and upper bound (the best integer solution) for each instance considered. We proposed a population based ant colony optimization (ACO) where the ants are subdivided into subpopulations and each subpopulation represents one inventory level to construct the routes. In addition, we modify the standard ACO by including the inventory cost in the global pheromones updating and the selection of inventory updating mechanism is based on the pheromone value. ACO performs better on large instances compared to the upper bound and performs equally well for small and medium instances.

Keywords: Inventory Routing Problem, Multi Products, Ant Colony Optimization

1 INTRODUCTION

Inventory routing problem (IRP) is classified as a NP-hard problem that involves the integration and coordination of inventory management and transportation, which the customers are relying on a central supplier to deliver the commodity on a repeated basis. The main objective of this problem is to minimize the corresponding costs (fixed and variable costs) in order to make the deliveries to customers on time. IRP indeed arise the attention in past years and there are many meta-heuristic methods have been developed, such as genetic algorithms, tabu search and branch-and-cut algorithms to suit the problems that lead to optimal or near optimal solutions. We will first present some literatures on IRP concentrating on the most recent literatures.

Federgruen and Zipkin [13] were among the first to study the IRP. The problem was treated as a single day problem with a limited amount of inventory and the customers' demands are assumed to be a random variable. The problem decomposes into a nonlinear inventory allocation problem which determines the inventory and shortage costs and a Travelling

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Table 1: Results of ACOBF and ACOPher

Data	LB (Objective)	UB (Best Integer)		ACOBf				ACOPher			
		Costs	# veh	Best Costs	#veh	Time (secs)	Gap* (%)	Best Costs	#veh	Time (secs)	Gap* (%)
S12T5	2033	2231.96	19	2285.94	19	21	2.42	2279.62	19	21	2.14
S12T10	4047.64	4305.33	36	4441.23	36	38	3.16	4436.01	36	38	3.04
S12T14	5329.58	6196.35	52	6422.24	52	51	3.99	6421.95	52	51	3.98
S20T5	3208.35	3394.78	28	3522.65	28	54	3.77	3527.80	28	55	3.92
S20T10	6330.97	6759.71	56	7046.23	56	102	4.24	7037.72	56	103	4.11
S20T14	8769.73	9368.08	77	9697.48	77	140	3.52	9689.11	77	140	3.43
S20T21	12407.58	13929.21	115	14487.10	113	207	4.01	14481.10	113	207	3.96
S50T5	7614.43	8213.22	64	8110.05	60	348	-1.26	8134.02	60	354	-0.96
S50T10	13913.84	17359.2	135	16871.20	124	712	-2.81	16852.70	125	721	-2.92
S50T14	19300.45	25181.61	197	23886.30	178	994	-5.14	23939.80	178	1013	-4.93
S50T21	29418.86	38626.96	311	36715.10	273	1509	-4.95	36572.00	274	1544	-5.32
S100T5	13208.54	16130.13	134	15080.20	122	1844	-6.51	15108.90	122	1892	-6.33
S100T10	25601.69	34388.15	293	30960.40	249	3698	-9.97	30857.20	249	3789	-10.27
S100T14	-	-	-	43997.50	355	5175		43934.60	355	5294	-

Salesman Problem (TSP) for each vehicle considered which produces the transportation costs. Most of the earlier works concentrates on an infinite planning horizon (see for example Aghezzaf et al. [1], Anily and Bramel [2] and Campbell and Savelsbergh [4]). However, Chien et al. [5] is amongst the first to simulate a multiple period planning model where the model is based on a single period approach. This is achieved by passing some information from one period to the next through inter-period inventory flow. Since then many researchers have focused their modeling on a finite planning horizon.

Coelho and Laporte [6] proposed branch-and-cut algorithm to solve multi product multi vehicle IRP with deterministic demand and stock out cost is not allowed. In this paper, Coelho and Laporte [6] implemented a solution improvement algorithm after branch-and-cut identifies a new best solution. The purpose of solution improvement algorithm is to approximate the cost of a new solution resulting from the vertex removal and reinsertions. In this paper, the authors also considered additional two features namely the driver partial consistency and visiting space consistency. The driver partial consistency plays the role of increasing the quality of the solution provided by the IRP both to customers and suppliers in a multi-product environment. The results show that the visiting space helps in reducing the search space while providing meaningful solution. This is the first paper that solves the problem with heterogeneous vehicles.

Dror and Trudeau [11] first introduced the split delivery VRP (SDVRP) by relaxing a constraint of the VRP that every customer is served by only one vehicle. The authors showed that the relaxation increased the flexibility of distribution and could lead to important savings, both in the total distance traveled and in the number of vehicles used. The SDVRP remains NP hard despite this relaxation (Dror and Trudeau [12]). Several authors (see for example Mjirda et al. [16], Moin et al. [17] and Yu et al. [21]) have extended the concept of split delivery in the multi-period IRP.

Moin et al. [17] had proposed an efficient hybrid genetic algorithm to solve the IRP in a many-to-one network which involves multi products where each supplier supplies different products and in multi period scenario. The problem of this paper is to find the minimum cost to pick up the products from a set of geographically dispersed suppliers over a finite planning horizon to assembly plant by using a fleet of capacitated homogeneous vehicles which housed at a depot and the split pick-ups are allowed. The proposed hybrid genetic algorithm is based on the allocation first route second strategy and takes both the inventory and the transportation costs (fixed and variable) into consideration. The computational experiments have been done on the data sets which were extended from the existing data sets to show the effectiveness of the proposed approach. Small, medium and large size problems are added to the existing data sets. With the increase of problem size, GA based algorithms performed relatively much better.

Nevertheless, the results obtained by Moin et al. [17] is improved by Mjirda et al. [16] through a two-phase Variable Neighborhood Search (VNS). The first phase developed the initial solution without considering the inventory while in the second phase the initial solution is iteratively improved using Variable Neighborhood Descent (VND) or a VNS algorithm. Linear Programming (LP) and a heuristic method which consider the priority of rules on suppliers and vehicles are developed to calculate the amount of products to collect from each supplier at each period during the planning horizon. Both algorithms are implemented using the proposed 7 neighborhood structures based on 3 elementary moves: Drop, add and change. The computational results show that the proposed methods give better results than the existing methods from the literature for both solution quality and the running time. Both Moin et al. [17] and Mjirda et al. [16] consider the many-to-one network, which is equivalent to one-to-many network under certain assumption.

Yu et al. [21] working on a large scale IRP that delivers a single product with split delivery and vehicle fleet size constraint. The problem is solved by using a Lagrangian relaxation method and it combines with the surrogate subgradient method. The solution of the model

company manufacturing chairs has four plants located around the country. The cost of manufacture, including raw material, per chair and the minimum and maximum monthly production for each plant is shown in the following table.

Plant	Cost per chair	Production	
		Maximum	Minimum
1	\$5	500	0
2	\$7	750	400
3	\$3	1000	500
4	\$4	250	250

Twenty pounds of wood is required to make each chair. The company obtains the wood from two sources. The sources can supply any amount to the company, but contracts specify that the company must buy at least eight tons of wood from each supplier. The cost of obtaining the wood at the sources is:

- Source 1 = \$0.10/pound
- Source 2 = \$0.075/pound

The shipping cost per pound of wood between each source and each plant is shown in cents by the following matrix:

		Plant			
		1	2	3	4
Wood	1	1	2	4	4
Source	2	4	3	2	2

The chairs are sold in four major cities: New York, Chicago, San Francisco and Austin. Transportation costs between the plants and the cities are shown in the following matrix: (All costs are in dollars per chair.)

		Cities			
		NY	A	SF	C
Plant	1	1	1	2	0
	2	3	6	7	3
	3	3	1	5	3
	4	8	2	1	4

The maximum and minimum demand and the selling price for the chairs in each city is shown in the following

City	Selling price	Demand	
		Maximum	Minimum
New York	\$20	2000	500
Austin	\$15	400	100
San Francisco	\$20	1500	500
Chicago	\$18	1500	500

obtained by the Lagrangian relaxation method is used to construct a near-optimal solution of the IRP by solving a series of assignment problems. Numerical experiments show that the proposed hybrid approach can find a high quality near-optimal solution for the IRP with up to 200 customers and 10 periods in a reasonable computation time.

Recently several researchers have applied ant colony optimization algorithm (ACO) metaheuristic in several variants of IRP. Huang and Lin [14] proposed a modified ACO for solving the multi-item inventory routing problems which the demand is stochastic by choosing a delivery policy that minimizes the total costs. The algorithm was developed for the replenishment of the vending machine and this modified ACO algorithm which incorporates the stock out cost in the calculation of the pheromone values which is not included in the conventional ACO. The nodes with high stock out costs are given higher priority even though the total transportation costs are higher than other nodes. The test instances were constructed using the Solomon's [19] 56 benchmark problems created for vehicle routing problem with time windows. The results show that the modified ACO algorithm achieves highly significant improvements compared to the conventional ACO.

Calvete et al. [3] is the first who studied a bilevel model in the context of hierarchical production-distribution (PD) planning. In this problem, a distribution company, which is the leader of the hierarchical process, controls the allocation of retailers to each depot and the routes which serve them. The manufacturing company which is the follower of the hierarchical process will decide which manufacturing plants will produce the orders received by the depot. ACO algorithm is developed to solve the bilevel model which ants are used to construct the routes of a feasible solutions for the associated multi-depot vehicle routing problem (MDVRP). The computational experiment is carried out to analyze the performance of the algorithm. Since the bilevel model is first time proposed, there is no data for comparison purposes. The computational time is reasonable, taking into account the problem sizes.

In the most recent work, Tatsis et al. [20] developed a mixed integer mathematical model where a fleet of capacitated homogeneous vehicle is used to deliver distinct products from multi suppliers to a retailer to meet the demand in each period over the planning horizon. However, backlogging is allowed in this study. The ant based optimisation algorithm is applied to solve the corresponding vehicle routing problem. The objective of this study is to find the best compromise between the transportation, inventory and backlogging cost. Preliminary results show that the solution gaps between the algorithm and CPLEX solutions is kept reasonably low values and offered prospective for further improvement.

In this study, we modify the formulation proposed by Yu et al. [21] to incorporate multi products. The network comprises of a warehouse that supplies multi products to a set of geographically dispersed customers by using a fleet of capacitated homogeneous vehicles and the deliveries must be complete on time. Customers may be visited more than once in each period and the demands are assumed to be relatively small as compared to the vehicle's capacity such that a coordinated delivery is more economically. We develop the ACO which was initially proposed by Dorigo and co-workers (Dorigo [7], Dorigo and Blum [9], and Dorigo and Caro [10]) and was inspired by the food-foraging behavior of ant to solve the proposed model. We proposed a population based ant colony optimization (ACO) where the ants are subdivided into subpopulations and each subpopulation represents one inventory level to construct the routes. The algorithm is modified by incorporating the inventory component in the global updating scheme that not only calculates the pheromone along the trail but identifies a set of feasible neighbours making use of the attractions on the nodes which differs from the classical ACO. In addition, the selection of inventory updating mechanism is based on the pheromone value.

This paper is organized as follows. Section 2 discusses problem formulation including the assumptions that are made in this model. The two modified ACO algorithms are described in



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detail in Section 3 and Section 4. It is followed by the computational results and discussion presented in Section 5. Finally, the conclusion is drawn in Section 6.

2 MODEL FORMULATION

We consider a one to many network where a fleet of homogeneous vehicle transports multi products from a warehouse or depot to a set of geographically dispersed customers in a finite planning horizon. The following assumptions are made in this model.

- The fleet of homogenous vehicles with limited capacity is available at the warehouse.
- Customers can be served by more than one vehicle (split delivery is allowed).
- Each customer requests a distinct product and the demand for the product is known in advance but may vary between different periods.
- The holding cost per unit item per unit time is incurred at the customer sites but not incurred at the warehouse. The holding cost does not vary throughout the planning horizon.
- The demand must be met on time and backordering or backlogging is not allowed.

The problem is modeled as mixed integer programming and the following notation is used in the model:

Indices

$\tau = \{1, 2, \dots, T\}$ period index

$W = \{0\}$ warehouse/depot

$S = \{1, 2, \dots, N\}$ a set of customers where customer i demands product i only

Parameters

C vehicles capacity (assume to be equal for all the vehicles).

F fixed vehicle cost per trip (assumed to be the same for all periods)

V travel cost per unit distance

M size of the vehicle fleet and it is assumed to be ∞ (unlimited)

c_{ij} travel distance between customer i and j where $c_{ij} = c_{ji}$ and the triangle inequality,

$c_{ik} + c_{kj} \geq c_{ij}$ holds for any i, j , and k with $i \neq j$, $k \neq i$ and $k \neq j$

h_i inventory carrying cost at the customer for product i per unit product per unit time

d_{it} demand of customer i in period t

Variables

a_{it} delivery quantity to customer i in period t

I_{it} inventory level of product i at the customer i at the end of period t

q_{ijt} quantity transported through the directed arc (ij) in period t

x_{ijt} number of times that the directed arc (ij) is visited by vehicles in period t

The model for our inventory routing problem is given as below:

$$Z = \min \underbrace{\sum_{t \in \tau} \sum_{i \in S} h_i I_{it}} + V \underbrace{\left(\sum_{t \in \tau} \sum_{j \in S} \sum_{i \in S \cup W} c_{ij} x_{ijt} + \sum_{t \in \tau} \sum_{i \in S} c_{i0} x_{i0t} \right)} + F \underbrace{\sum_{t \in \tau} \sum_{i \in S} x_{i0t}} \quad (1)$$

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subject to

$$I_{it} = I_{i,t-1} + a_{it} - d_{it}, \forall i \in S, \forall t \in \tau \quad (2)$$

$$\sum_{\substack{j \in S \cup W \\ i \neq j}} q_{ijt} + a_{it} = \sum_{\substack{j \in S \cup W \\ i \neq j}} q_{jit}, \forall i \in S, \forall t \in \tau \quad (3)$$

$$\sum_{i \in S} q_{0it} = \sum_{i \in S} a_{it}, \forall t \in \tau \quad (4)$$

$$\sum_{\substack{i \in S \cup W \\ i \neq j}} x_{ijt} = \sum_{\substack{i \in S \cup W \\ i \neq j}} x_{jit}, \forall j \in W, \forall t \in \tau \quad (5)$$

$$I_{it} \geq 0, \forall i \in S, \forall t \in \tau \quad (6)$$

$$a_{it} \geq 0, \forall i \in S, \forall t \in \tau \quad (7)$$

$$q_{ijt} \geq 0, \forall i \in S \cup W, \forall j \in S, j \neq i, \forall t \in \tau \quad (8)$$

$$q_{ijt} \leq Cx_{ijt}, \forall i \in S \cup W, \forall j \in S, i \neq j, \forall t \in \tau \quad (9)$$

$$x_{ijt} \in \{0,1\}, \forall i, j \in S, \forall t \in \tau \quad (10)$$

$$x_{0jt} \geq 0, \text{ and integer}, \forall j \in S, \forall t \in \tau \quad (11)$$

The objective function (1) includes the inventory costs (I), the transportation costs (II) and vehicle fixed cost (III). (2) is the inventory balance equation for each product at the warehouse while (3) is the product flow conservation equations, to ensure that the flow balance at each customer and eliminating all subtours. (4) assures the collection of accumulative delivery quantity at the warehouse (split delivery). (5) ensures that the number of vehicles leaving the warehouse is equal to the number of vehicles returning to warehouse. (6) assures that the demand at the warehouse is completely fulfilled without backorder. Meanwhile, (9) guarantees that the vehicle capacity is respected and gives the logical relationship between q_{ijt} and x_{ijt} which allows for split delivery. This formulation is used to determine the lower and upper bounds for each data set using CPLEX 12.4.

3 POPULATION BASED ACO

Ant Colony Optimization (ACO) inspired by the nature behavior of ants finding the shortest path between their colony and a source of food to solve the proposed model of inventory routing problem. The information collected by ants during the searching process is stored in pheromone trails. Hence, when an ant has built a solution, the ant deposits a certain amount of pheromone proportionally (the information about the goodness of the solution) on the pheromone trails of the connection it used. The pheromone information directs search of the following ants while exploring the graph. The higher density of pheromones on an arc leads to attract more ants to the arc. Therefore, an appropriate formulation associated to the model for updating pheromones trail is very crucial. This is due to the reason that the greater amounts of pheromone it deposits on the arcs tend to provide a shorter path (the minimum cost).

In this study, we modified the classical ACO that comprise a single population to several subpopulations. Instead of having one population (where the ants all have the same inventory) we subdivide the ants into subpopulations. Each subpopulation represents one inventory level. Although we started with the same inventory level for all the subpopulations, we observed that after a few iterations the inventory for each subpopulation differs greatly between each other. Consequently the pheromone value differs between subpopulation as we select the best solution according to the subpopulation to generate the

pheromone value. Note that the ant shares the same pheromone value within the same subpopulation.

In the classical ACO, the routing factor is taken into account to build the routing part of the algorithms. However, our algorithm also incorporates the inventory cost in the formulation of global updating in order to balance between the transportation and inventory cost. The procedure for ACO can be divided into three main steps: the route construction, a local pheromone-update rule and a global pheromone-update rule. These steps are described in detail in the following subsections and the following definitions are required:

- τ_0 the amount of pheromone deposited (an initial pheromone value assigned to all arcs).
- Q a random number generated from a uniform distribution over the interval (0, 1).
- q_0 a predefined real number where $0 \leq q_0 \leq 1$.
- τ_{ij} refer to the pheromone value allocated on arc (i, j) .
- $\eta_{ij} = \frac{1}{c_{ij}}$ where c_{ij} is the length of arc (i, j) .
- α, β the parameters to control the influence of the pheromone value allocated on arc (i, j) and the desirability of arc (i, j) respectively.
- Ω_i all the arcs connected to unvisited node j (unvisited customer) such that the ants in node i passing through arc (i, j) will not violate any constraint.

3.1 Initial Solution

We construct the initial solution by having all the demand met in every period. In this study we adopt a simple Nearest Neighbour algorithm (NN) and the algorithm is modified to allow for split delivery. The vehicle starts at the depot and repeatedly visits the nearest customer (in terms of distance) until the capacity of the vehicle is fully occupied. Then, a new vehicle is initiated and the process continues until all customers have been assigned or visited. The total distance obtained by NN is embedded to initialize the τ_0 , the initial pheromone in the local pheromone updating. We have divided the updating of the solutions into two parts: Routing and Inventory. The updating mechanism involves the local pheromone and global pheromone updating.

3.2 Route Construction

The route construction begins by setting the value of all the parameters $\alpha, \beta, \tau_0, q_0$ and ρ . α, β are two parameters that control the influence of the pheromone value allocated on arc (i, j) and the desirability of arc (i, j) respectively whilst q_0 is a predefined real number where $0 \leq q_0 \leq 1$ and ρ is the rate of pheromone evaporations. Note the value of τ_0 , the initial value of pheromones for each arc is obtained from the total distance of the initial solution.

The algorithm starts with the predefined number of subpopulation of ants and each subpopulation consists of predefined number of ants to build the solution. Starting from the depot (warehouse) each ant utilizes equation (12) to select the next customer to be visited. Ants tend to be attracted to the arc which consists of higher density of pheromones. From equation (12), if q is less than the predefined parameter q_0 , then the next arc chosen is the arc with the highest attraction. Otherwise, the next arc is chosen using the biased Roulette Method with the state transition probability p_{ij} given by equation (14).

$$j = \begin{cases} \max_{j \in \Omega_i} \{Att_{i,j}\} & \text{if } q \leq q_0 \\ p_{ij} & \text{otherwise} \end{cases} \quad (12)$$

$$\text{where } Att_{ij} = (\tau_{ij})^\alpha (\eta_{ij})^\beta \quad (13)$$

$$p_{ij} = \begin{cases} \frac{(\tau_{ij})^\alpha (\eta_{ij})^\beta}{\sum_{k \in \Omega_i} (\tau_{ik})^\alpha (\eta_{ik})^\beta} & \forall j \in \Omega_i \\ 0 & \forall j \notin \Omega_i \end{cases} \quad (14)$$

τ_{ij} is the amount of pheromone deposited on arc(i, j) and η_{ij} is inversely proportional to the length of arc (i, j), c_{ij} . Ω_i a set of unvisited customers for ant i . Figure 1 illustrates the algorithm. Note that the inventory updating mechanism is further elaborated in Subsection 3.3.2.

Step1: Start the algorithm with Nearest Neighbour Algorithm and obtained the total distance.

Do the following steps from $i = 1$ to $i = \text{MaxIter}$,

Step 2: Do the route construction by using the ACO for all the ants in each subpopulation.

Step 2.1: Local pheromones updating

Choose the best solution among all the ants in each subpopulation to do the local pheromones updating.

Step 2.2: Route improvement strategies

Choose the best solution among all the subpopulations to do the route improvement strategies.

Step 2.3: Global pheromones Updating

IF ($i \bmod \text{PredefinedIterationForGL}$) = 0, then

Choose the current best built solution to do the global pheromones updating. Then, go to Step 3.

Else, go to Step 3.

Step 3: Update inventory level mechanism

IF ($i \bmod \text{predefinedIterInvUpt}$) = 0

Update the inventory for all subpopulation except the subpopulation containing the current best solution.

Go to Step 2.

Else, go to Step 2.

Figure 1: Algorithm of Population Based ACO

3.2.1 Local Pheromones Updating

Local updating is used to reduce the amount of pheromone on all the visited arcs in order to simulate the natural evaporation of pheromone and it is intended to avoid a very strong arc being chosen by all the ants. After each of the ants in every subpopulation has built the solution, the best built solution from each subpopulation will be selected. Then, the local updating will be done on each arc of the best solution from each subpopulation by using equation (15).

(15)

$$\tau_{ij} = (1 - \rho)\tau_{ij} + (\rho)\tau_0$$

where ρ represents the rate of pheromone evaporation.

3.2.2 Global Pheromones Updating

After a predefined number of iterations, the current best solution γ^{gl} among all the subpopulations is selected and its routes are used as a reference for the global pheromones-updating for all subpopulation. Hence, the pheromones value for each arc of the best solution is updated by using the equation (16) for all the subpopulations. The global pheromone-updating rule resets the ant colony's situation to a better starting point and encourages the use of shorter routes. Moreover, it increases the probability that future routes use the arcs contained in the best solutions. In the classical ACO, only the transportation cost is taken into account in the global updating. Since the IRP tries to find a balanced between the transportation and inventory cost, it is natural to incorporate the inventory holding cost in the formulation. The global update rule is enhanced as follows:

(16)

$$\tau_{ij} = (1 - \rho)\tau_{ij} + \frac{\rho}{J_{\gamma^{gl}}}(i, j) \in \gamma^{gl}$$

where $J_{\gamma^{gl}}$ is the weight of the best solution found where it incorporates the inventory element as well as the variable transportation costs and is given by

(17)

$$J_{\gamma^{gl}} = \sum_{i \in I} \sum_{j \in S} h_i I_{ij} + V \left(\sum_{i \in I} \sum_{j \in S} \sum_{k \in S} c_{ijk} x_{ijk} + \sum_{i \in I} \sum_{j \in S} c_{i0j} x_{i0j} \right)$$

3.2.3 Route improvement strategies

The routes can be further improved by adding route improvement strategies in the route construction procedure. In this study, there are three local searches: swap, 2 - opt* and 2 - opt are applied to improve the solution built by ACO.

Swap

The first local search is the swap algorithm focusing on the split customers and they comprise of a transfer to the selected vehicle or a swap between different vehicles. Starting from the last vehicle, the split customer is identified and we try to merge to the current selected vehicle if the respective vehicle capacity is not violated. If this fails, then the swap with the other customers from the preceding vehicle or to the current selected vehicle that results in the least transportation cost is carried out. If none of the swap provides an improvement in the objective value than the route remains unchanged. The process continues until all vehicles in every period have been examined. The aim of this method is to eliminate the split customers (merge as many as possible) if the merged improved the objective value.

2-opt*

We applied 2 - opt* [18] heuristic as inter route optimization procedure. The purpose of this strategy is to test on all possible pair wise exchange between vehicles to see if an overall

improvement in the objective function can be attained. The heuristic calculates the distances for all pair wise permutations and compared those distances with the current solution. If any of these solutions is found to improve the objective function, then it replaces the current solution.

2-opt

2-opt [15] heuristic which is an intra-route optimization procedure is utilized to improve the route within the same vehicle. The implementation is slightly different from 2-opt*, 2-opt test on all possible pair wise exchange within a vehicle instead of between vehicles to see if an overall improvement in the objective function can be obtained. The current solution is replaced if the improved solution is better.

3.3 Updating Inventory Level

After the predefined number of iterations, the inventory updating mechanism will be initiated. We propose two types of transfers, the forward and backward transfers to update the inventory level. The selection of the forward and backward transfer is controlled by a random number but the emphasis is more towards the forward transfer. The selection of their quantity delivery based on the predefined number. If the current number of iteration is less than the predefined number, then the customer will be selected randomly. Otherwise, the customer who fulfilled the conditions (highest and lowest inventory holding cost for the forward and backward transfer respectively) will be selected. We observed that implementing a combination of randomly generated and deterministically generated customers allow the ants to have more explorations at the beginning of the iterations.

In our first algorithm we observe that the algorithm is more likely to give savings in term of transportation costs instead of inventory. We modified the algorithm such that the inventory updating is based on the pheromone value of each customer. The following equation is used to update the pheromones of customer's inventory for customer j in period p , $invpher_{jp}$:

$$invpher_{jp} = (1 - \rho) * (invpher_{jp}) + \rho * \frac{a_{jp}}{c_{ij}} \quad (18)$$

a_{jp} is the quantity delivery of customer j in period p while c_{ij} is the distance between the customer i and j for the routing of the current best solution and ρ represents the rate of pheromone evaporation as defined earlier. The second element of equation (18) specifies that if the ratio of the delivery quantity of the respective customer to its distance is higher, then higher pheromone value will be allocated. The selection of c_{ij} for local and global inventory is discussed in more details in the following subsections.

3.3.1 Local pheromones updating of customer's inventory

For the local inventory updating the c_{ij} value is taken from the best solution after some predefined number of iterations. Note that the predefined number of iterations is smaller than the predefined value of the overall inventory updating. The purpose of selecting the current best solution after a few iterations instead of selecting the best from all the current built solutions is to avoid the customer with strong customer's inventory pheromones being chosen all the time by all the subpopulations.

3.3.2 Global pheromones updating of customer's inventory

A slightly different mechanism is used in selecting the value of c_{ij} for the global inventory updating. The value of c_{ij} is selected from the current best solution. The current best solution is chosen among all the current built. The aim of this step is to guide the ants to select the best amount of quantity delivery to be carried for the customer. Figure 2

illustrates the algorithm for the process of inventory updating. Note that $predefinedIterInvUptGL < predefinedIterInvUpt < PredefinedIterationForGL_p$, and

$$PredefinedIterationForGL_p = \frac{predefinedIterInvUpt}{PredefinedIterationForGL}$$

For $i = 1$ to $i = \text{MaxIter}$

Do the following steps after the route construction of ACO.

IF $i \bmod predefinedIterInvUpt = 0$, then

IF $i \bmod predefinedIterInvUptGL = 0$, then

Choose the current best solution among all the built solutions.

Use the routing part of the best solution to update the customer's inventory pheromones (Global pheromones updating of customer's inventory)

Then, continue with the updating inventory mechanism.

Else

Choose the best solution from $PredefinedIterationForGL_p$ solutions that have been built.

Use the routing part of the best solution to update the customer's inventory pheromones (Local pheromones updating of customer's inventory)

Then, continue with the updating inventory mechanism.

Else

Continue with the route construction part.

Figure 2: Algorithm of updating customer's inventory pheromones for the second modified algorithm of ACO

3.3.3 Procedure of updating inventory level for each subpopulation

After the customer's inventory pheromones have been updated, the algorithm proceeds to select the customer to undergo the transfer. As mentioned earlier, the mechanism of selecting customer is based on the random number that has been generated but the priority is given to the customer based on the attraction. If random value is less than certain predefined parameter, the customer with the highest attraction of inventory pheromones is selected to do the undergo the transfer. The attraction of customer's inventory pheromones for customer j in period p , $AttInv_{jp}$ in each subpopulation is calculated using the equation

$$AttInv_{jp} = (invPher_{jp})^\mu * (1/a_{jp})^\omega \quad (19)$$

Otherwise, the deterministic backward / forward transferring mechanism (highest and lowest inventory holding cost for the forward and backward transfer respectively) will be used to select the customer to update the inventory.

4 COMPUTATIONAL RESULTS

The algorithms were written in C++ language by using Microsoft Visual studio 2008. The results of this study is compared with the lower bound (LB) and the upper bound (UB) generated by solving the formulation presented in Section 3 using CPLEX 12.4. All the computations were performed on 3.10 GHz processor with 8GB of RAM.

4.1 Data sets

The algorithm is tested on 12, 20, 50 and 100 customers, and combination with different number of periods, 5, 10, 14 and 21. The coordinates for each customer is generated randomly in the square of 100×100 . The coordinates of each customer for the 20 customer instance comprises the existing 12 customer instance with additional 8 newly randomly generated coordinates. The same procedure is used to create the 50 and 100 customer instances. The holding cost for each customer lies between 0 and 10 while the demand for each of the customer is generated randomly between 0 and 50. The vehicle capacity is 100.

4.2 Results and Discussion

For all the instances, we let CPLEX 12.4 run for a limited time 9000s (2.5 hours) in order to obtain the lower bound and the best integer solution. The results of ACOBF which implemented the forward as well as backward transferring in the mechanism of updating inventory level and results of ACOPher which implemented the customer's inventory pheromones are shown in Table 1. The parameters for both ACOBF and ACOPher are set as follows: $\alpha = 1.0$, $\beta = 5.0$, $q_0 = 0.9$, $\rho = 0.1$, $\tau_0 = 1/L_{nn}$ where L_{nn} is the total distance obtained from nearest neighbor algorithm. The values of α , β and q_0 are taken from Dorigo et al.[8]. The pheromones on customers's inventory in ACOPher is initially set as $1/d_{ip}$ if d_{ip} is not equal to 0, otherwise set as 0. In both ACOBF and ACOPher, there are 5 subpopulations and each subpopulation consists of 5 ants to build solution.

Table1 show the results of the population based ACO, ACOBF and the population based ACOthat includes the inventory pheromones, ACOPher. Table 1 presents the best costs, the number of vehicles, the CPU time, the lower bound and the upper bound (best integer solutions) which are obtained from CPLEX. From Table 1, we observed that the gaps which is calculated as the ratio of the difference between the lower bound and the upper bound to the lower bound, for all the solutions are greater than 10%. This ratio increases as the periods and the number of customers increase. Thus, it is hard to justify the quality of the lower bound obtained by CPLEX. This may due to the lower bound is really loose or the upper bound is rather poor.

From the results show in Table 1 for both ACOBF and ACOPher, we note that the total costs of the data sets with 100 and 50 customers are less than the upper bound which means the algorithm is able to obtain better results when compared with the upper bound for 50 and 100 customer instances. However, both algorithms perform equally well for the small and medium instances and produced the gaps between the results and the best integer solutions that are less than 4.5 percent. The ACOPher perform better in 10 out of 14 instances.

5 CONCLUSION

The integration of inventory and transportation plays an important role in supply chain management. In this study we present the formulation of the model that consists of multi-products and multi-periods IRP as well as the development of a modified ACO to construct the routes. We design a population based ACO by segregating each subpopulation by the inventory level. We have constructed a modified global routing updating that includes some information on inventory. A new transfer/swap aims at split customers is also proposed. This is carried out in order to collapse some the vehicles and to reduce the overall transportation cost.

A new inventory updating mechanism is also proposed where it takes into account the customers inventory in the local and global pheromone updating. The selection of customers for the transfer is based on the attraction. We found that, by including the selection based on the attraction, the results better in almost all instances.